



## 53 - SEQUENCES

### Finding the *n*th Term of a Linear Sequence

This method works for **linear sequences** — ones with a **common difference** (where the terms **increase** or **decrease** by the **same amount** each time). Linear sequences are also known as **arithmetic sequences**.

#### EXAMPLE:

Find an expression for the *n*th term of the sequence that starts 5, 8, 11, 14, ...

n:	1	2	3	4
term:	5	8	11	14
		+3	+3	+3
3n:	3	6	9	12
	↓+2	↓+2	↓+2	↓+2
term:	5	8	11	14

So the expression for the *n*th term is  $3n + 2$

The common difference is 3, so '3n' is in the formula.

You have to + 2 to get to the term.

- 1) **Find the common difference** — this tells you what to multiply *n* by. So here, 3 gives '3n'.
- 2) **Work out what to add or subtract.** So for *n* = 1, '3n' is 3 so add 2 to get to the term (5).
- 3) **Put both bits together.** So you get  $3n + 2$ .

Always **check** your expression by putting the first few values of *n* back in, e.g. putting *n* = 1 into  $3n + 2$  gives 5, *n* = 2 gives 8, etc. which is the **original sequence** you were given — hooray!

### Finding the *n*th Term of a Quadratic Sequence

A **quadratic sequence** has an  $n^2$  term — the **difference** between the terms **changes** as you go through the sequence, but the **difference** between the **differences** is the **same** each time.

#### EXAMPLE:

Find an expression for the *n*th term of the sequence that starts 10, 14, 20, 28...

n:	1	2	3	4
term:	10	14	20	28
		+4	+6	+8
		+2	+2	
term:	10	14	20	28
$n^2$ :	1	4	9	16
term - $n^2$ :	9	10	11	12

So the expression for the *n*th term is  $n^2 + n + 8$

So the expression will contain an  $n^2$  term.

The expression for this linear sequence is  $n + 8$

- 1) Find the **difference** between each pair of terms.
- 2) The difference is **changing**, so work out the difference between the **differences**.
- 3) **Divide** this value by **2** — this gives the coefficient of the  $n^2$  term (here it's  $2 \div 2 = 1$ ).
- 4) **Subtract** the  $n^2$  term from each term in the sequence. This will give you a **linear sequence**.
- 5) Find the **rule** for the *n*th term of the linear sequence (see above) and **add** this on to the  $n^2$  term.

$N^{\text{th}}$  term of a Quadratic Sequences is Higher only

Other Types of sequences: Some sequences are generated by doing something with the previous term(s) for example you may need to add the previous 2 terms together to get the next term. 1, 1, 2, 3, 5, 8... (1 + 1 = 2) (1 + 2 = 3) (2 + 3 = 5) etc This is the Fibonacci sequence.

#### Linked Prior Topics

Solving equations, basic sequences, finding the term to term rule, writing sequences given the rule term to term rule, writing a sequence given the *n*th term rule.

#### Vocabulary

Linear sequence      Quadratic sequence  
Nth term              Difference

#### Linked Future Topics

Using sequences to solve problems.