



## 57 - Solving Quadratic Equations

### Content

When we solve quadratic equations, there will be two solutions. There are 3 algebraic ways to solve quadratic equations.

You can also solve quadratic equations using a graph, find the points where the graph crosses the x-axis, these are your solutions.

#### Method 1 – Factorisation

First we factorise the quadratic into 2 brackets. As at least one of these brackets must equal 0. We then have 2 linear equations to solve.

#### Example

Solve  $x^2 - 2x - 15 = 0$

Factorise into 2 brackets

$$(x + 3)(x - 5) = 0$$

Either

$$(x + 3) = 0 \text{ or } (x - 5) = 0$$

Therefore

$$x = -3 \text{ or } x = 5$$

#### Method 2 – Completing the Square (Higher only)

This method can be used when we can't easily factorise the quadratic and usually has surds in the solutions.

#### Example

Solve  $x^2 - 4x - 3 = 0$

Rearrange so that the unknowns are on one side

$$x^2 - 4x = 3$$

Halve the coefficient of  $x$ . This number must be put into a bracket, along with  $x$ , and squared. We then subtract the square of this number

$$(x - 2)^2 - 4 = 3$$

Solve the equation

$$(x - 2)^2 = 7$$

$$x - 2 = \pm\sqrt{7}$$

$$x = 2 + \sqrt{7} \text{ or } x = 2 - \sqrt{7}$$

Remember that the square root of a number can be either positive or negative.

#### Method 3 – The Quadratic Formula (higher only)

This method is also used when we cannot easily factorise the quadratic. This will be on a calculator paper and the answers will generally be decimals.

#### Formula

For quadratic equations of the form

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ax^2 + bx + c = 0$$

#### Example

Solve  $2x^2 + 11x - 5 = 0$ . Give your answer to 2 decimal places.

$$a = 2, b = 11, c = -5$$

Substitute these into the quadratic formula, use brackets for negative numbers.

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

Put this into the calculator, first with a + and then with a - to find your two solutions.

$$x = -5.92 \text{ or } x = 0.42$$

### Linked Prior Topics

Factorising quadratic equations, solving equations, substituting into formulae

### Vocabulary

Equation, quadratic, factorise, formula, substitute, coefficient

### Linked Future Topics

Problem solving, sketching graphs