Hillcrest Mathematics Knowledge Organiser

7 - Algebraic Proof

Content

- To demonstrate or show that a statement is true, we use examples.
- To prove that a statement is true you can use algebra.

Some useful generalisations

Consecutive Integers	n, n + 1, n + 2,…
Even Numbers	2n
Odd Numbers	2n + 1
Consecutive Evens	2n, 2n + 2, 2n + 4,…
Consecutive Odd	2n +1, 2n + 3,….

Prove that the square of an odd number is always odd.

Let the odd number be $2n + 1$.

So $(2n+1)^2 = (2n+1)\,(2n+1)$
$= 4n^2 + 4n + 1$
$= 2\,(2n^2 + 2n) + 1$

As $2(2n^2 + 2n)$ is even, then when we add 1, the number must be odd.

The sum of any three consecutive integers is a multiple of 3.

Let the consecutive integers be $n, n+1\ and\ n+2$.

So $n + (n+1) + (n+2) = 3n + 3$
$= 3(n+1)$

This is a multiple of 3.

Prove for any 3 consecutive integers the difference between the product of the first 2 and the product of last two is always twice the middle number.

Let the consecutive integers be $n, n+1\ and\ n+2$.

The product of the first and second

$$n\,(n+1) = n^2 + n$$

The product of the second and third

$$(n+1)\,(n+2) = n^2 + 3n + 2$$

So the difference between these products is

$$(n^2 + 3n + 2) - (n^2 + n) = 2n + 2$$

This equals $2\,(n+1)$ which is twice the middle number.

Linked Prior Topics

Equations, simplifying, expanding brackets, factorising

Vocabulary

Algebraic, proof, consecutive, product, difference, multiple

Linked Future Topics

Proof by induction, proof by contradiction