Hillcrest Mathematics Knowledge Organiser

29 - Indices, roots, reciprocals and Hierarchy of operations

Indices are the numbers written above a base number. E.g. For 8^2 – 8 is the base number and 2 is the index

The **index** shows how many times it has been multiplied by itself. E.g. 3^7 = 3x3x3x3x3x3x3

1) When MULTIPLYING, you ADD THE POWERS.
e.g. $3^6 \times 3^4 = 3^{6+4} = 3^{10}$, $a^2 \times a^7 = a^{2+7} = a^9$

2) When DIVIDING, you SUBTRACT THE POWERS.
e.g. $5^4 \div 5^2 = 5^{4-2} = 5^2$, $b^8 \div b^5 = b^{8-5} = b^3$

3) When RAISING one power to another, you MULTIPLY THEM.
e.g. $(3^2)^4 = 3^{2\times4} = 3^8$, $(c^3)^6 = c^{3\times6} = c^{18}$

4) $x^1 = x$, ANYTHING to the POWER 1 is just ITSELF.
e.g. $3^1 = 3$, $d \times d^3 = d^1 \times d^3 = d^{1+3} = d^4$

5) $x^0 = 1$, ANYTHING to the POWER 0 is just 1.
e.g. $5^0 = 1$, $67^0 = 1$, $e^0 = 1$

6) $1^x = 1$, 1 TO ANY POWER is STILL JUST 1.
e.g. $1^{23} = 1$, $1^{89} = 1$, $1^2 = 1$

7) FRACTIONS — Apply the power to both TOP and BOTTOM.
e.g. $\left(1\frac{3}{5}\right)^3 = \left(\frac{8}{5}\right)^3 = \frac{8^3}{5^3} = \frac{512}{125}$, $\left(\frac{u}{v}\right)^5 = \frac{u^5}{v^5}$

Hierarchy of operations

B (Brackets)
I Indices ^{2} Can also be known as O -Other
D Division ÷
M Multiplication x
A Addition +
S Subtraction –

Reciprocal

The reciprocal of a number is 1 ÷ the number

e.g. The reciprocal of 5 is $\frac{1}{5}$.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

Warning!
Rules 1 & 2 don't work for different bases: $2^3 \times 3^7$

EXAMPLE: Simplify $(3a^2b^4c)^3$

Just deal with each bit separately:

$= (3)^3 \times (a^2)^3 \times (b^4)^3 \times (c)^3$
$= 27 \times a^{2\times3} \times b^{4\times3} \times c^3$
$= 27a^6b^{12}c^3$

8) NEGATIVE Powers — Turn it Upside-Down

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: "Aha, that means turn it the other way up and make the power positive".

e.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $a^{-4} = \frac{1}{a^4}$, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{+2} = \frac{5^2}{3^2} = \frac{25}{9}$

9) FRACTIONAL POWERS

The power $\frac{1}{2}$ means Square Root,
The power $\frac{1}{3}$ means Cube Root,
The power $\frac{1}{4}$ means Fourth Root etc.

e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$
$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$
$z^{\frac{1}{5}} = \sqrt[5]{z}$

The one to really watch is when you get a negative fraction like $49^{-1/2}$ — people get mixed up and think that the minus is the square root, and forget to turn it upside down as well.

10) TWO-STAGE FRACTIONAL POWERS

With fractional powers like $64^{\frac{5}{6}}$ always split the fraction into a root and a power, and do them in that order: root first, then power: $(64)^{\frac{1}{6}\times5} = (64^{\frac{1}{6}})^5 = (2)^5 = 32$.

EXAMPLE: Find the reciprocal of $\sqrt{4+6\times(12-2)}$.

$\sqrt{4+6\times(12-2)} = \sqrt{4+6\times10}$ ← It's not obvious what to do inside the square root — so use BODMAS. Brackets first...
$= \sqrt{4+60}$ ← ... then multiply...
$= \sqrt{64}$ ← ... then add.
$= 8$ ← Take the square root
The reciprocal of 8 is $\frac{1}{8}$. ← Finally, take the reciprocal (the reciprocal of a number is just 1 ÷ the number).

Linked Prior Topics
- Adding and subtracting (inc. negative numbers)
- Multiplication

Vocabulary
- Indices
- Powers = indices
- Exponent = Indices
- Base number - number with an index

Linked Future Topics
- Standard form
- Surds
- Quadratics